

The singular fibers of pencils of curves, The application of modern mathematics



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Keywords	Complex Geometry, Mathematical Physics
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Technical Support Skills	<ul style="list-style-type: none"> • I can give some advices about the meaning of equations which appear in physics and engineering science. • something about algebraic curves • something about complex manifolds
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Research Contents the singular fibers of pencils of curves, the application of modern mathematics

★ We have studied the degeneration of curves, especially the singular fibers of pencils of curves.

Let $\pi: X \rightarrow D$ be a proper surjective holomorphic map of a complex manifold X of dimension 2 to a small open disc $D = \{t \in \mathbb{C} \mid |t| < \epsilon\}$. We assume that π is smooth over a punctured disc $D' = D - \{0\}$. Moreover we assume that for every $t \in D'$ the fiber $X_t = \pi^{-1}(t)$ is a non-singular curve of genus g and that X contains no exceptional curves of the first kind. By L_t we denote the effective divisor in X defined by the equation $\pi = t$ ($t \in D$). We call the divisor L_0 the singular fiber of π . For every $t \in D'$ we call the divisor L_t a generic fiber. We write the singular fiber L_0 as

$$L_0 = \sum_{i=1}^r n_i \Gamma_i,$$

where Γ_i is an irreducible reduced component of L_0 and n_i is its multiplicity. By $p(\Gamma_i)$ we denote the arithmetic genus of the component Γ_i . The combination of integers $\{r, n_i, p(\Gamma_i), \Gamma_i \cdot \Gamma_j \ (1 \leq i < j \leq r)\}$ is called a numerical type of the singular fiber L_0 .

In the study of elliptic surfaces Kodaira showed that there exist only ten types of singular fibers of pencils of curves of genus one. Itaka and Ogg gave a numerical classification of singular fibers of curves of genus 2. Namikawa and Ueno classified their numerical types completely, constructed all their singular fibers and calculated the monodromies around them.

We gave a method to classify all the numerical types of singular fibers of pencils of curves of genus g ($g \geq 2$). We got some numerical properties of singular fibers, such as the inequality $s \leq 12(g - 1)$, where s is the largest number of multiplicities in the singular fiber. By using these method we classified the numerical types of singular fibers of pencils of genus 3 curves.

We will proceed to study the singular fibers and the surface with these singular fibers.

★ We are also interested in the application of modern mathematics, such as differential forms or tensor to physics and engineering.

Available Facilities and Equipment
